

## MAT 175 Fall 09 Quiz 9: Solution

1) a) Since  $\begin{bmatrix} 1 & 2 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ , we can conclude that

the rank of  $\begin{bmatrix} 1 & 2 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix}$  is 3 and  $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \right\}$

is a basis of  $\mathbb{R}^3$ .

b) Set  $c_1 \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} + c_3 \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \\ 1 \end{bmatrix}$ .

Since  $\left[ \begin{array}{ccc|c} 1 & 2 & 2 & 6 \\ -1 & 1 & 0 & -3 \\ 0 & -1 & 0 & 1 \end{array} \right] \sim \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array} \right]$ , we can

conclude that  $c_1 = 2$ ,  $c_2 = -1$  and  $c_3 = 3$ . Moreover,

$$[\vec{u}]_B = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}.$$

$$2) a) [T]_{\mathcal{B}} = \begin{bmatrix} 3 & -1 & 5 \\ -2 & 3 & -2 \\ 1 & 0 & -1 \end{bmatrix}.$$

b) The standard matrix of  $T$  is

$$\begin{aligned} & B[T]_{\mathcal{B}} B^{-1} \\ &= \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 3 & -1 & 5 \\ -2 & 3 & -2 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & 2 & 2 \end{bmatrix}^{-1} \\ &= \begin{bmatrix} 17 & 6 & -7 \\ -19 & -7 & 9 \\ 15 & 6 & -5 \end{bmatrix}. \end{aligned}$$

$$c) T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 17x + 6y - 7z \\ -19x - 7y + 9z \\ 15x + 6y - 5z \end{bmatrix}.$$