

MAT 175 Fall 2009: Quiz 4

Dr. Gaywalee Yamskulna

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Name:.....SOLUTION.....

Instruction: Please read the questions carefully. You must write complete solutions to receive complete credit.

1. (4 points) Let $A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$, and let $C = \begin{bmatrix} 3 & 8 & 1 \\ 2 & 0 & 4 \end{bmatrix}$. Find $C^T A$.

Solution

$$C^T A = \begin{bmatrix} 3 & 2 \\ 8 & 0 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3(-1) + 2(3) & 3(2) + 2(4) \\ 8(-1) + 0(3) & 8(2) + 0(4) \\ 1(-1) + 4(3) & 1(2) + 4(4) \end{bmatrix} = \begin{bmatrix} 3 & 14 \\ -8 & 16 \\ 11 & 18 \end{bmatrix}.$$

2. (6 points) Let

$$\begin{aligned} x_1 + x_2 + x_3 &= -5 \\ 2x_1 + x_2 + x_3 &= -3 \\ 3x_1 &+ x_3 = 2 \end{aligned}$$

be system of linear equations.

- (a) Write the system as a matrix equation $A\vec{x} = \vec{b}$.

Solution $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 3 & 0 & 1 \end{bmatrix} \vec{x} = \begin{bmatrix} -5 \\ -3 \\ 2 \end{bmatrix}$.

- (b) Show that A is invertible, and find A^{-1} .

Solution

Since $[A|I_3]$ is row equivalent to $\begin{bmatrix} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 2 & -1 \\ 0 & 0 & 1 & 3 & -3 & 1 \end{bmatrix}$, we can

conclude that A is invertible and $A^{-1} = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 2 & -1 \\ 3 & -3 & 1 \end{bmatrix}$.

- (c) Use A^{-1} to solve the system.

Solution $\vec{x} = A^{-1}\vec{b} = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 2 & -1 \\ 3 & -3 & 1 \end{bmatrix} \begin{bmatrix} -5 \\ -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ -4 \end{bmatrix}$.