

MAT 175 Fall 2009: Quiz 3

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Name:.....SOLUTION.....

Instruction: Please read the questions carefully. You must write complete solutions to receive complete credit.

1. (5 points) Determine whether the set $\left\{ \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix} \right\}$ is a generating set for \mathbb{R}^3 .

Solution

Since the reduce row echelon form of the matrix $A = \begin{bmatrix} -1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 3 & 1 \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, we can conclude that $\text{Rank } A = 2$. Since $\text{Rank } A \neq 3$, it follows that the given set is not a generating set for \mathbb{R}^3 .

2. (5 points) Determine, if possible, a value of r for which the given set $\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 9 \\ r \\ 2 \end{bmatrix} \right\}$ is linearly dependent.

Solution

To determine the values r , we will apply sequences of row operations on the matrix $A = \begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & -1 & 1 & 9 \\ -1 & 2 & 1 & r \\ 1 & 1 & 0 & 2 \end{bmatrix}$. First we apply row operations $R_3 + R_1 \mapsto R_3$ and $R_4 - R_1 \mapsto R_4$ on the matrix A . We obtain a matrix $A_2 = \begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & -1 & 1 & 9 \\ 0 & 2 & 0 & r-1 \\ 0 & 1 & 1 & 3 \end{bmatrix}$. Next, we apply row operations $R_3 + 2R_2 \mapsto R_3$, $R_4 + R_2 \mapsto R_4$ on A_2 . We then ob-

tain a matrix $A_3 = \begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & -1 & 1 & 9 \\ 0 & 0 & 2 & r+17 \\ 0 & 0 & 2 & 12 \end{bmatrix}$. Finally, we apply row operation $R_4 - R_3 \mapsto R_4$ on a matrix A_3 . We then have a matrix $A_4 = \begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & -1 & 1 & 9 \\ 0 & 0 & 2 & r+17 \\ 0 & 0 & 0 & -r-5 \end{bmatrix}$. The matrix A_4 is a row echelon form of the matrix A .

Notice that the given set $\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 9 \\ r \\ 2 \end{bmatrix} \right\}$ is linearly dependent when the last row of the matrix A_4 is a zero row. Hence when $r = -5$, the given set is linearly dependent.