

MAT 175 Fall 2009: Quiz 2

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Name:.....SOLUTION.....

Instruction: Please read the questions carefully. You must write complete solutions to receive complete credit.

- (7 points) Determine the values of r and s for which the given system of linear equations has
 - no solutions,
 - exactly one solution, and
 - infinitely many solutions.

$$\begin{aligned}2x_1 - x_2 &= 3 \\4x_1 + rx_2 &= s.\end{aligned}$$

Solution The augmented matrix of the given system is $\left[\begin{array}{cc|c} 2 & -1 & 3 \\ 4 & r & s \end{array} \right]$. By using row operation $R_2 - 2R_1 \mapsto R_2$ on a such matrix, we obtain that the augmented matrix is row equivalent to a matrix $\left[\begin{array}{cc|c} 2 & -1 & 3 \\ 0 & r+2 & s-6 \end{array} \right]$.

Answer for a) The system has no solution when the last column of an equivalent row echelon form is a pivot column. Hence, the system has no solution when $r + 2 = 0$ and $s - 6 \neq 0$. So, the values of r and s for which the given system has no solution are $r = -2$ and $s \neq 6$.

Answer for b) The system has a unique solution when the first and second columns are pivot position. Hence, the system has a unique solution when $r + 2 \neq 0$. So, the values of r and s for which the given system has a unique solution are $r \neq -2$ and s is any real number.

Answer for c) The system has infinitely many solutions when the second row is a zero row. Hence, the system has infinitely many solutions when $r + 2 = 0$ and $s - 6 = 0$. So, the values of r and s for which the given system has infinitely many solutions are $r = -2$ and $s = 6$.

2. (3 points) Find the rank and nullity of the matrix $\begin{bmatrix} 1 & 0 & 1 & -1 & 6 \\ 2 & -1 & 5 & -1 & 7 \\ -1 & 1 & -4 & 1 & -3 \\ 0 & 1 & -3 & 1 & 1 \end{bmatrix}$.

Solution

The reduced row echelon form of the matrix $\begin{bmatrix} 1 & 0 & 1 & -1 & 6 \\ 2 & -1 & 5 & -1 & 7 \\ -1 & 1 & -4 & 1 & -3 \\ 0 & 1 & -3 & 1 & 1 \end{bmatrix}$

is $\begin{bmatrix} 1 & 0 & 1 & 0 & 4 \\ 0 & 1 & -3 & 0 & 3 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

Since there are 3 pivot columns and this matrix has 5 columns, we can conclude that the rank of the given matrix is 3 and the nullity is $5 - 3 = 2$.