MAT 175 Fall 2009: Quiz 1
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Name:.............SOLUTION...

Instruction: Please read the questions carefully. You must write complete solutions to receive complete credit.

1. Let $A = \begin{bmatrix} 1 & -1 & 5 \\ 3 & 7 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 5 & 4 \end{bmatrix}$. Find $(3A - \frac{1}{2}B)^T$. Please show your work in detail. (5 points)

   Solution Since

$$\begin{aligned} 3A - \frac{1}{2}B &= \begin{bmatrix} 3 & -3 & 15 \\ 9 & 21 & 3 \end{bmatrix} - \begin{bmatrix} 1/2 & 1/2 & -1/2 \\ 1 & 5/2 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 5/2 & -7/2 & 31/2 \\ 8 & 37/2 & 1 \end{bmatrix}, \end{aligned}$$

   we then have that $(3A - \frac{1}{2}B)^T = \begin{bmatrix} 5/2 & 8 \\ -7/2 & 37/2 \\ 31/2 & 1 \end{bmatrix}$.

2. Assume that

   (a) $\vec{u}$ is a linear combination of the vectors $\vec{v}_1,\ \vec{v}_2,\ \vec{v}_3$, and

   (b) $\vec{v}_1,\ \vec{v}_2$ and $\vec{v}_3$ are linear combinations of the vectors $\vec{w}_1,\ \vec{w}_2,\ \vec{w}_3,\ \vec{w}_4$.

   Show that $\vec{u}$ is also a linear combination of $\vec{w}_1,\ \vec{w}_2,\ \vec{w}_3,\ \vec{w}_4$. (5 points)

   Solution

   Since $\vec{u}$ is a linear combination of the vectors $\vec{v}_1,\ \vec{v}_2,\ \vec{v}_3$, it follows that there exist scalars a, b, c such that

$$\vec{u} = a\vec{v}_1 + b\vec{v}_2 + c\vec{v}_3. \qquad (0.1)$$

   Since $\vec{v}_1,\ \vec{v}_2$ and $\vec{v}_3$ are linear combinations of the vectors $\vec{w}_1,\ \vec{w}_2,\ \vec{w}_3,\ \vec{w}_4$, we have that there exist scalars $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, c_1, c_2, c_3, c_4$ such that

$$\begin{aligned} \vec{v}_1 &= a_1\vec{w}_1 + a_2\vec{w}_2 + a_3\vec{w}_3 + a_4\vec{w}_4, & (0.2) \\ \vec{v}_2 &= b_1\vec{w}_1 + b_2\vec{w}_2 + b_3\vec{w}_3 + b_4\vec{w}_4, & (0.3) \\ \vec{v}_3 &= c_1\vec{w}_1 + c_2\vec{w}_2 + c_3\vec{w}_3 + c_4\vec{w}_4. & (0.4) \end{aligned}$$

Substitute equations (0.2)-(0.4) into an equation (0.1), we can conclude that

$$\begin{aligned}\vec{u} = & \quad a(a_1\vec{w}_1 + a_2\vec{w}_2 + a_3\vec{w}_3 + a_4\vec{w}_4) \\ & + \quad b(b_1\vec{w}_1 + b_2\vec{w}_2 + b_3\vec{w}_3 + b_4\vec{w}_4) \\ & + \quad c(c_1\vec{w}_1 + c_2\vec{w}_2 + c_3\vec{w}_3 + c_4\vec{w}_4).\end{aligned}$$

Hence, $\vec{u}$ is a linear combination of $\vec{w}_1, \vec{w}_2, \vec{w}_3, \vec{w}_4$.