

3) Proof Set $u_1 = \text{l.u.b } S_1$ and $u_2 = \text{l.u.b } S_2$ and $S_1 + S_2 = \{x+y \mid x \in S_1, y \in S_2\}$.

Step 1 We will show that $u_1 + u_2$ is an upper bound of $S_1 + S_2$.

Since $x \leq u_1, y \leq u_2$ for all $x \in S_1, y \in S_2$, we can conclude that

$x+y \leq u_1 + u_2$ for all $x \in S_1$ and $y \in S_2$. Hence, $u_1 + u_2$ is an upper bound of $S_1 + S_2$.

Step 2 We will show that $u_1 + u_2$ is the least upper bound of $S_1 + S_2$.

Let $u \in \mathbb{R}$ s.t. $u < u_1 + u_2$. Since $u - u_2 < u_1$, we can conclude that $u - u_2$ is not an upper bound of S_1 . Hence $\exists x \in S_1$ s.t. $u - u_2 < x$ ($\Rightarrow u - x < u_2$).

Since $u - x < u_2$, it implies that $u - x$ is not an upper bound of S_2 .

Hence, $\exists y \in S_2$ s.t. $u - x < y$. Consequently, $u < x + y$. So,

u is not an upper bound of $S_1 + S_2$. Therefore $u_1 + u_2$ is l.u.b of $S_1 + S_2$.