MAT 347 Spring 2010: Quiz 2
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Name: Solution

Instruction: Please read the questions carefully. You must write complete solutions to receive complete credit.

1. Prove in detail that for any $a, b, c, d \in \mathbb{R}$

$$(a-b)(c-d) = (ac+bd) - (ad+bc).$$

2. Show that if $a, b, x, y \in \mathbb{R}$ and $a < x < b$, $a < y < b$ then $|x-y| < b-a$.

1) $(a-b)(c-d) = (a-b)(c+(-d)) = (a-b)c + (a-b)(-d)$
$= (a+(-b))c + (a+(-b))(-d)$
$= ac + (-b)c + a(-d) + (-b)(-d)$
$= ac + (-1)(bc) + (-1)ad + bd$
$= (ac+bd) + (-1)(bc+ad)$
$= (ac+bd) - (bc+ad)$

2) Let $a, b, x, y \in \mathbb{R}$ s.t $a<x<b$, $a<y<b$.
WTS $|x-y| < b-a$. Recall that $|x-y| = \begin{cases} x-y & \text{if } x \geq y \\ y-x & \text{if } x<y. \end{cases}$ (***)

Since $x<b$ and $y>a$, we then have that $b-x \in \mathbb{R}_+$ and $y-a \in \mathbb{R}_+$. Furthermore, we have $b-a+y-x = b-x+y-a \in \mathbb{R}_+$. Therefore, $b-a > x-y$. (*).
Since $y<b$ and $x>a$, it follows that $b-y \in \mathbb{R}_+$ and $x-a \in \mathbb{R}_+$. Hence, $(b-a)+(x-y) \in \mathbb{R}_+$. Consequently, $b-a > y-x$. (**)

By (*), (**), (***), we can conclude that $|y-x| < b-a$.