Clearly $(0,1) \in \complement S_6$.

If $r \geq 1$ then $N((0,1), r) = \{(x,y) \mid x^2 + (y-1)^2 < r^2\} \ni (0, \frac{1}{2})$.

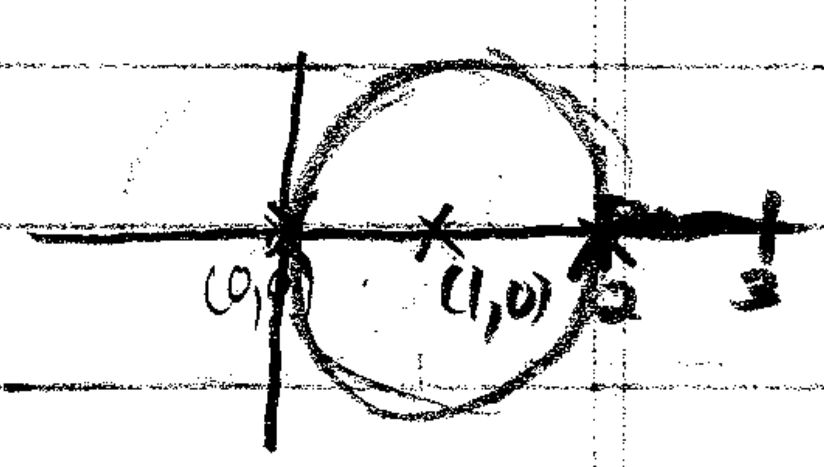
Since $(0, \frac{1}{2}) \in S_6$, we can conclude that $N((0,1), r) \not\subset \complement S_6$.

If $r < 1$ then $N((0,1), r) \ni (0, 1-\frac{r}{2})$. Since $(0, 1-\frac{r}{2}) \in S_6$, we can conclude that $N((0,1), r) \not\subset \complement S_6$.

Reason: $\frac{r}{2} < r < 1 \Rightarrow 1-\frac{r}{2} > 0$ and $-\frac{r}{2} < 0 \Downarrow 1-\frac{r}{2} < 1$

Hence $\complement S_6$ is not open and S_6 is not closed.

7) Set $S_7 = \{(x,y) \mid x^2 - 2x + y^2 = 0\} \cup \{(x,0) \mid x \in [2,3]\}$



$= \{(x,y) \mid x^2 - 2x + 1 + y^2 = 1\} \cup \{(x,0) \mid x \in [2,3]\}$

$= \{(x,y) \mid (x-1)^2 + y^2 = 1\} \cup \left(\{(x,y) \mid 2 \leq x \leq 3,\ 0 \leq y \leq 1\} \cap \{(x,y) \mid 2 \leq x \leq 3,\ -1 \leq y \leq 0\}\right)$

Since $\{(x,y) \mid (x-1)^2 + y^2 = 1\}$ is a sphere of center $(1,0)$ and radius 1, we can conclude that it is a closed set. $(*)$

Since $\{(x,y) \mid 2 \leq x \leq 3,\ 0 \leq y \leq 1\}$ and $\{(x,y) \mid 2 \leq x \leq 3,\ -1 \leq y \leq 0\}$ are closed sets, it follows that

$\{(x,0) \mid x \in [2,3]\} = \{(x,y) \mid 2 \leq x \leq 3,\ 0 \leq y \leq 1\} \cap \{(x,y) \mid 2 \leq x \leq 3,\ -1 \leq y \leq 0\}$

is a closed set. $(**)$

By $(*)$, $(**)$, we can conclude that S_7 is closed.

S_7 is not open. Reason: Clearly $(0,0) \in S_7$. Let $r > 0$. Since $N((0,0), r) \ni (0, -\frac{r}{2})$ and $(0, -\frac{r}{2}) \notin S_7$, we can conclude that $N((0,0), r) \not\subset S_7$