1) Find a generating set for the range of the linear transformation.

$T: \mathbb{R}^3 \to \mathbb{R}^2$ defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 2x_2 \\ x_2 - x_3 \end{bmatrix}$

2) Find a generating set for the null space of the linear transformation.

$T: \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 \\ 0 \\ 2x_1 - x_2 \end{bmatrix}$.

3) Find the standard matrix of a linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^2$

defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 2x_2 \\ x_2 - x_3 \end{bmatrix}$ and use it to determine whether

T is one-to-one.

4) Find the standard matrix of a linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^2$

defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 + x_3 \\ x_1 + x_2 - x_3 \end{bmatrix}$ and use it to determine

whether T is onto.

<u>Solution</u>

1) $\text{Range}(T) = \text{Span}\left\{ T(\vec{e}_1), \text{[illegible]}, T(\vec{e}_3) \right\}$

$= \text{Span}\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \text{[illegible]} \right\}.$

Hence a generating set is [illegible].

Note: [illegible] is also a generating set.

2) $\text{Null}(T) = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \;\middle|\; T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

$= \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \;\middle|\; \begin{bmatrix} x_1 + x_2 \\ 0 \\ 2x_1 - x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

$= \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \;\middle|\; \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

$= \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

$$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 2 & -1 & 0 \end{array}\right] \xrightarrow{R_3 - 2R_1} \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & -3 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 / 3} \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right] \xrightarrow{R_1 - R_3} \begin{array}{c} \begin{array}{cc} x_1 & x_2 \\ \downarrow & \downarrow \end{array} \\ \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right] \end{array}$$

A generating set is $\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$.

Hence T is 1-1.

3) The standard matrix is $\begin{array}{c} \begin{array}{ccc} T(\vec{e}_1) & T(\vec{e}_2) & T(\vec{e}_3) \\ \downarrow & \downarrow & \downarrow \end{array} \\ \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & -1 \end{bmatrix} \end{array}$. Since the rank of the standard matrix is 2 $(\neq 3)$, we can conclude that T is not 1-1.

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4) The standard matrix is $\begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix}$. The rank is 2. Hence T is onto.