MAT 175 Spring 2010: Quiz 9
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Name: Solution

Instruction: Please read the questions carefully. You must write complete solutions to receive complete credit.

1. (5 points) Find a basis and the dimension of the solution space of the homogeneous system of linear equations

$$\begin{aligned} 4x - y + 2z &= 0 \\ 2x + 3y - z &= 0 \\ 3x + y + z &= 0. \end{aligned}$$

2. (5 points) Let W be the subspace of $\mathbb{R}^4$ spanned by the vectors $\begin{bmatrix} 1 \\ -2 \\ 5 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 3 \\ 1 \\ -4 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ 8 \\ -3 \\ -5 \end{bmatrix}$.

   (a) Find a basis and the dimension of W.

   (b) Extend the basis of W to a basis of the whole space $\mathbb{R}^4$.

1) Solution space $= \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \;\middle|\; \begin{matrix} 4x - y + 2z = 0, \\ 2x + 3y - z = 0, \\ 3x + y + z = 0 \end{matrix} \right\} = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \;\middle|\; \begin{bmatrix} 4 & -1 & 2 \\ 2 & 3 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

$= \text{Null}\left(\begin{bmatrix} 4 & -1 & 2 \\ 2 & 3 & -1 \\ 1 & 1 & 1 \end{bmatrix} \right)$

Since $\begin{bmatrix} 4 & -1 & 2 \\ 2 & 3 & -1 \\ 1 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, we can conclude that $x = 0$, $y = 0$, and $z = 0$.

Hence Solution space $= \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$. Since the number of free variables is zero, we can conclude that the dimension of solution space is zero, and a basis of the solution space is an empty set.

2) Since $\begin{bmatrix} 1 & 2 & 3 \\ -2 & 3 & 8 \\ 5 & 1 & -3 \\ -3 & -4 & -5 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, we can conclude that

$$W = \text{Span}\left\{ \begin{bmatrix} 1 \\ -2 \\ 5 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 1 \\ -4 \end{bmatrix}, \begin{bmatrix} 3 \\ 8 \\ -3 \\ -5 \end{bmatrix} \right\} = \text{Col}\left(\begin{bmatrix} 1 & 2 & 3 \\ -2 & 3 & 8 \\ 5 & 1 & -3 \\ -3 & -4 & -5 \end{bmatrix} \right) = \text{Span}\left\{ \begin{bmatrix} 1 \\ -2 \\ 5 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 1 \\ -4 \end{bmatrix} \right\}$$

and a basis of W is $\left\{ \begin{bmatrix} 1 \\ -2 \\ 5 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 1 \\ -4 \end{bmatrix} \right\}$, and $\dim W = 2$.

Since $\begin{bmatrix} 1 & 2 & 0 & 0 \\ -2 & 3 & 0 & 0 \\ 5 & 1 & 1 & 0 \\ -3 & -4 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, we can conclude that

$\left\{ \begin{bmatrix} 1 \\ -2 \\ 5 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ is a basis of $\mathbb{R}^4$.