

Name: Solution

Instruction: Please read the questions carefully. You must write complete solutions to receive complete credit.

1. Let $A = \begin{bmatrix} 1 & -1 & 5 \\ 3 & 7 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 5 & 4 \end{bmatrix}$, and $\vec{v} = \begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix}$. Find

(a) $(3A - \frac{1}{2}B)^T$. (3 points)

(b) $A(4\vec{v})$. (3 points)

2. Let $\vec{u} = \begin{bmatrix} 2 \\ -2 \\ 8 \end{bmatrix}$ and let $S = \left\{ \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix} \right\}$. If possible, write \vec{u} as a linear combination of the vectors in S . (4 points)

$$\begin{aligned} 1) a) 3A - \frac{1}{2}B &= 3 \begin{bmatrix} 1 & -1 & 5 \\ 3 & 7 & 1 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} 1 & 1 & -1 \\ 2 & 5 & 4 \end{bmatrix} = \begin{bmatrix} 3 & -3 & 15 \\ 9 & 21 & 3 \end{bmatrix} - \begin{bmatrix} 1/2 & 1/2 & -1/2 \\ 1 & 5/2 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 5/2 & -7/2 & 3/2 \\ 8 & 37/2 & 1 \end{bmatrix} \end{aligned}$$

Hence, $(3A - \frac{1}{2}B)^T = \begin{bmatrix} 5/2 & 8 \\ -7/2 & 37/2 \\ 3/2 & 1 \end{bmatrix}$

b) $A(4\vec{v}) = \begin{bmatrix} 1 & -1 & 5 \\ 3 & 7 & 1 \end{bmatrix} \begin{bmatrix} 8 \\ 16 \\ 20 \end{bmatrix} = 8 \begin{bmatrix} 1 \\ 3 \end{bmatrix} + 16 \begin{bmatrix} -1 \\ 7 \end{bmatrix} + 20 \begin{bmatrix} 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 92 \\ 156 \end{bmatrix}$

2) Set $x_1 \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 8 \end{bmatrix}$. Hence, we have that $\begin{matrix} -x_2 = 2, (\Rightarrow x_2 = -2) \\ x_1 + 3x_2 = -2, \text{ and} \\ 2x_1 = 8. (\Rightarrow x_1 = 4) \end{matrix}$

Substitute $x_1 = 4$ and $x_2 = -2$ into the eq $x_1 + 3x_2 = -2$ we have that $4 + 3(-2) = -2$. Therefore, $\begin{bmatrix} 2 \\ -2 \\ 8 \end{bmatrix}$ is a lin. comb of $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}$.