MAT 175 Spring 2010: Quiz 11
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Name: Solution

Instruction: Please read the questions carefully. You must write complete solutions to receive complete credit.

1. (5 points) Let $\vec{u} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$.
   (a) Compute the norm of each of the vectors.
   (b) Compute the distance d between the vectors.
   (c) Compute the dot product of the vectors.
   (d) Determine whether the vectors are orthogonal.

2. (5 points) Show that if $\vec{u}$ and $\vec{v}$ are orthogonal nonzero vectors in $\mathbb{R}^n$ then they are linearly independent.

1) a) $\|\vec{u}\| = \sqrt{2^2+(-3)^2} = \sqrt{4+9} = \sqrt{13}$, $\|\vec{v}\| = \sqrt{4^2+1^2} = \sqrt{17}$

b) $d = \|\vec{u}-\vec{v}\| = \left\| \begin{bmatrix} 2 \\ -3 \end{bmatrix} - \begin{bmatrix} 4 \\ 1 \end{bmatrix} \right\| = \left\| \begin{bmatrix} -2 \\ -4 \end{bmatrix} \right\| = \sqrt{(-2)^2+(-4)^2} = \sqrt{4+16} = \sqrt{20}$

c) $\vec{u}\cdot\vec{v} = \begin{bmatrix} 2 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 1 \end{bmatrix} = 2(4)+(-3)1 = 8-3 = 5$

d) since $\vec{u}\cdot\vec{v} \neq 0$, we can conclude that $\vec{u}$ and $\vec{v}$ are not orthogonal.

2) Let $\vec{u},\vec{v}$ be orthogonal nonzero vectors in $\mathbb{R}^n$.
Set $c_1\vec{u}+c_2\vec{v}=\vec{0}$.
<u>Goal</u> We will show that $c_1=c_2=0$.
Since $\vec{v}\cdot(c_1\vec{u}+c_2\vec{v}) = \vec{v}\cdot\vec{0} = 0$ and $\vec{v}\cdot(c_1\vec{u}+c_2\vec{v}) = c_1\vec{v}\cdot\vec{u}+c_2\vec{v}\cdot\vec{v}$,
~~and~~ we then have that $c_1\vec{v}\cdot\vec{u}+c_2\|\vec{v}\|^2=0$. Since $\vec{v}\cdot\vec{u}=0$,
it implies that $c_2\|\vec{v}\|^2=0$. Since $\|\vec{v}\|^2\neq 0$, it follows that $c_2=0$.
So, $\vec{0}=c_1\vec{u}+c_2\vec{v} = c_1\vec{u}$. Since $\vec{u}\neq\vec{0}$, we then have that $c_1=0$.

Therefore, $\vec{u},\vec{v}$ are linearly independent.